The Divergence Theorem

Recall we studied volume integrals of the form:

$$\iiint_V g(\bar{r})\, dv$$

It turns out that **any** and **every** scalar field can be written as the divergence of some **vector** field, i.e.:

$$g(\bar{r}) = \nabla \cdot \mathbf{A}(\bar{r})$$

Therefore we can equivalently write any volume integral as:

$$\iiint_V \nabla \cdot \mathbf{A}(\bar{r})\, dv$$

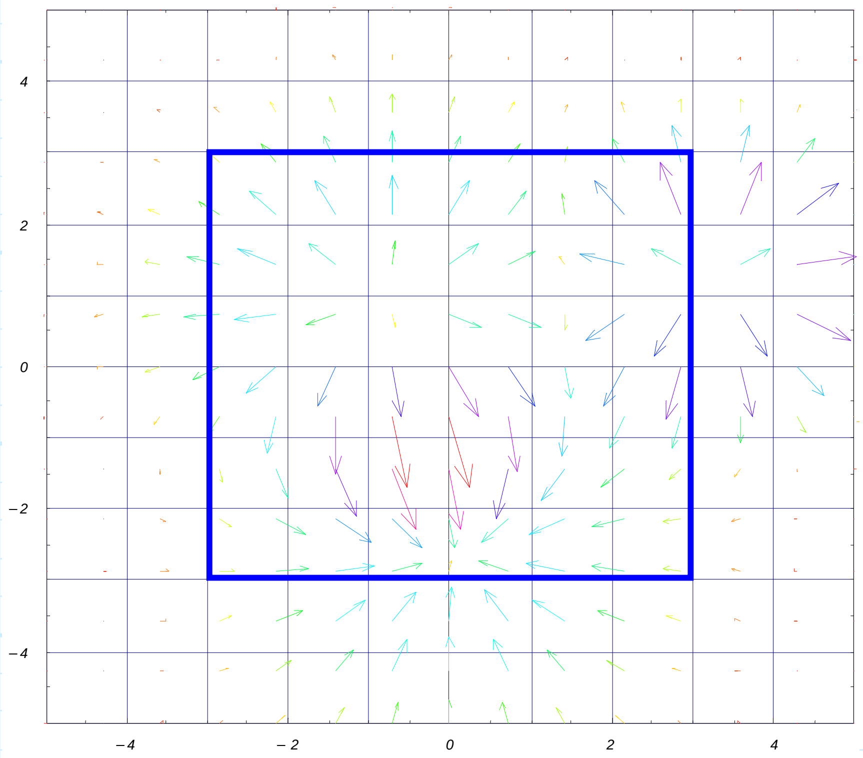
The **divergence theorem** states that this integral is equal to:

$$\iiint_V \nabla \cdot \mathbf{A}(\bar{r})\, dv = \oiint_S \mathbf{A}(\bar{r}) \cdot \overline{ds}$$

where S is the **closed** surface that completely surrounds volume V, and vector $\overline{ds}$ points **outward** from the closed surface. For example, if volume V is a **sphere**, then S is the **surface** of that sphere.

The divergence theorem states that the **volume** integral of a scalar field can be likewise evaluated as a **surface** integral of a vector field!

What the divergence theorem indicates is that the **total** "divergence" of a vector field through the **surface** of any volume is equal to the sum (i.e., integration) of the divergence at **all points** within the **volume**.



In other words, if the vector field is **diverging** from some point in the volume, it must simultaneously be **converging** to another adjacent point within the volume—the net effect is therefore **zero**!

Thus, the only values that make **any** difference in the **volume integral** are the divergence or convergence of the vector field across the surface surrounding the volume—vectors that will be converging or diverging to adjacent points **outside** the volume (across the surface) from points **inside** the volume. Since these points just outside the volume are not included in the integration, their net effect is **non-zero**!